Homework 6

In this HW you are **NOT** allowed to use the Church-Turing thesis.

Question 1. [15 marks] Show that every finite subset of $\mathbb{N}$ is primitive recursive. [*Hint.* Show that singleton sets are primitive recursive, first.]

Question 2. [25 marks] Show that the function:

$$\begin{aligned} f : \mathbb{N} &\to \mathbb{N} \\ 0 &\mapsto 1 \\ 1 &\mapsto 1 \\ n+2 &\mapsto f(n) + f(n+1), \end{aligned}$$

is primitive recursive.

Exercise 3. [60 marks] For each of the following functions, construct a register machine that computes it:

(1) $\lambda x.x^2$.

(2) $\lambda xy.x \times y$.

Question 4. [0 marks, Challenge!] Recall that *Euler's constant* is (sometimes) defined to be:

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} + \cdots.$$

Show that the function that on input n computes the n-th digit of e is primitive recursive.